5.1 (CONTINUED)

Transforming an equation from Polar to Rectangular Form.

Example 8 on p.324

Transform $\mathbf{r = 4\sin\theta}$ to rectangular coordinates.

Since $x^2 + y^2 = r^2$ multiply both sides by r to get r^2 on the left.

$r^2 = 4r\sin\theta$

Substitute $x^2 + y^2$ for r^2 and y for $r\sin\theta$

$x^2 + y^2 = 4y$

Put everything on the left to see this is an equation of a circle.

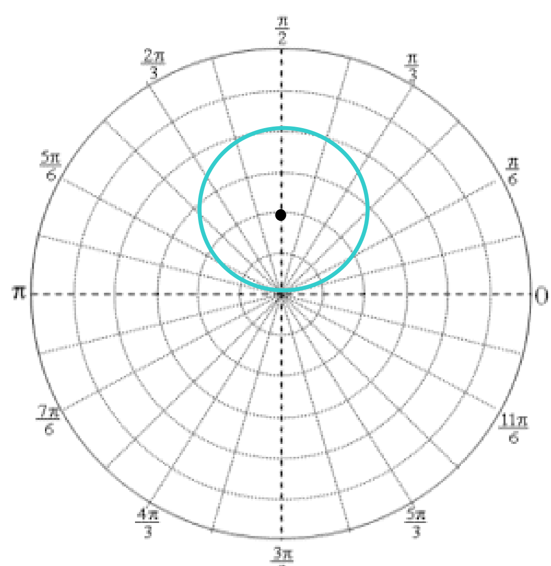
$x^2 + y^2 - 4y = 0$

Recall that we must complete the square to find out what the radius and center are. We do nothing to the x terms, since there is no "*ax*" term, but since "*by*" = -4y, add $(-4/2)^2$, which is 4, to both sides

$(x-0)^2 + (y^2 - 4y + 4) = 0 + 4$

$(x-0)^2 + (y - 2)^2 = 4 = 2^2$

So on the rectangular coordinate system, this is a circle with center (0,2) and radius = 2. On the polar grid, rectangular coordinate (0,2) translates to $(2, \pi/2)$.



Notice that the original polar form of the equation was $\mathbf{r = 4\sin\theta}$ and the center of the circle in rectangular coordinates is (0,2). Another way to write this equation is

$\mathbf{r = 2a\sin\theta}$ where a=2

You'll find that any polar equation in this form will be a circle with center at (0,a). *(We'll talk more about this later.)*

Example 9 <u>Transforming an equation from Rectangular to Polar Form</u>

$4xy = 9$

Remember, $x = r\cos\theta$ and $y = r\sin\theta$

$4(r\cos\theta)(r\sin\theta) = 9$

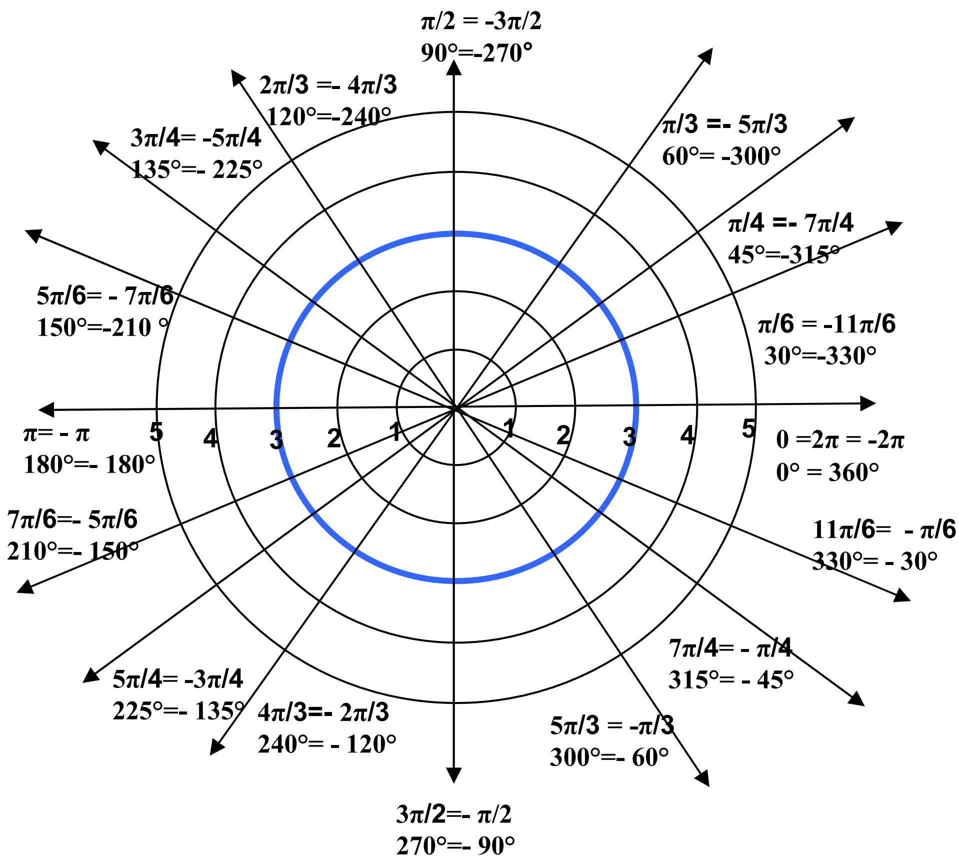
$4r^2\cos\theta\sin\theta = 9$

When you see $\sin\theta$ and $\cos\theta$ multiplied together this should remind you that $\sin(2\theta) = 2\sin\theta\cos\theta$, so rearranging our equation gives:

$2r^2(2\cos\theta\sin\theta) = 2r^2(2\sin\theta\cos\theta) = \underline{2r^2 \sin(2\theta) = 9}$

5.2 INDENTIFYING AND GRAPHING POLAR EQUATIONS

Example 1 Identify and graph $r = 3$.
There is no θ in this equation, which means θ can be anything. This is the graph of all the points at a distance $r = 3$ from the pole.
The set of all points equidistant from a center is a circle. Recall $r^2 = x^2 + y^2$, which is a circle if r is constant. In this case, $x^2 + y^2=9$



By the way, if you graph a circle using your calculator in rectangular points, you have to break it into 2 functions:

$$Y_1 = \sqrt{r^2 - x^2} \quad \text{and} \quad Y_2 = -\sqrt{r^2 - x^2}$$

(Upper semicircle) *(Lower semicircle)*

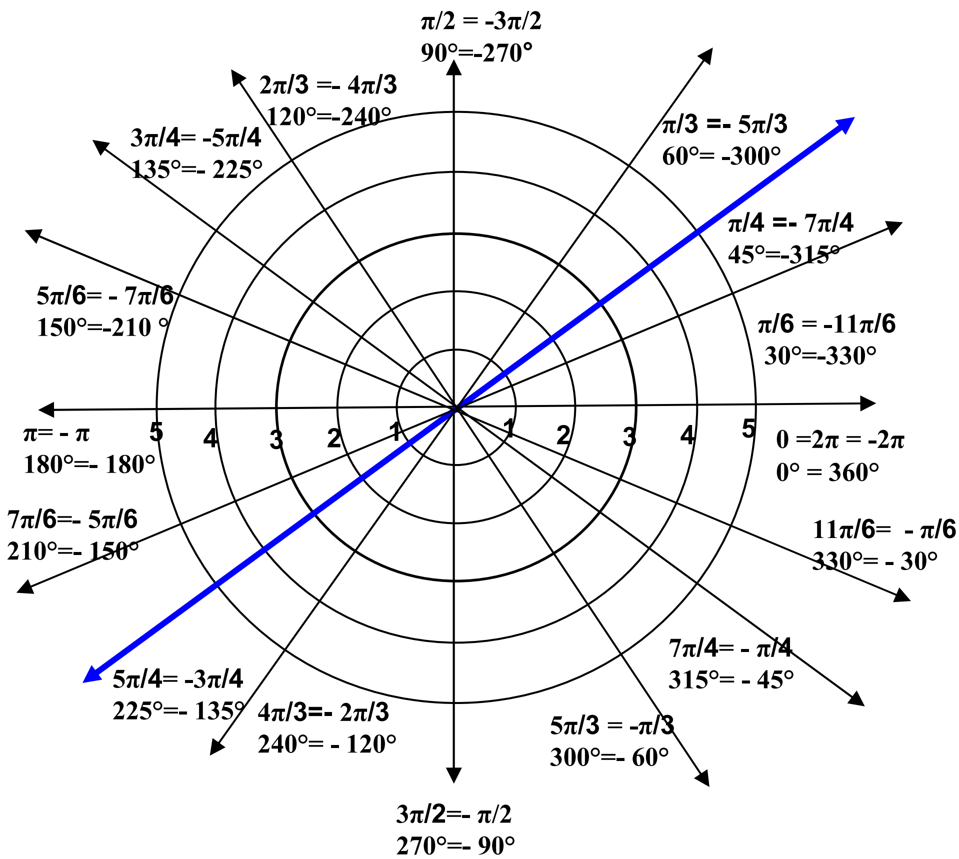
since $x^2 + y^2 = r^2$ is not a function, but in polar coordinates, it r=3 is a function, because for each choice of θ there is only one corresponding value or r, namely, r =3, so your calculator will have no problem.

Example 2

Identify and graph $\theta = \pi/4$

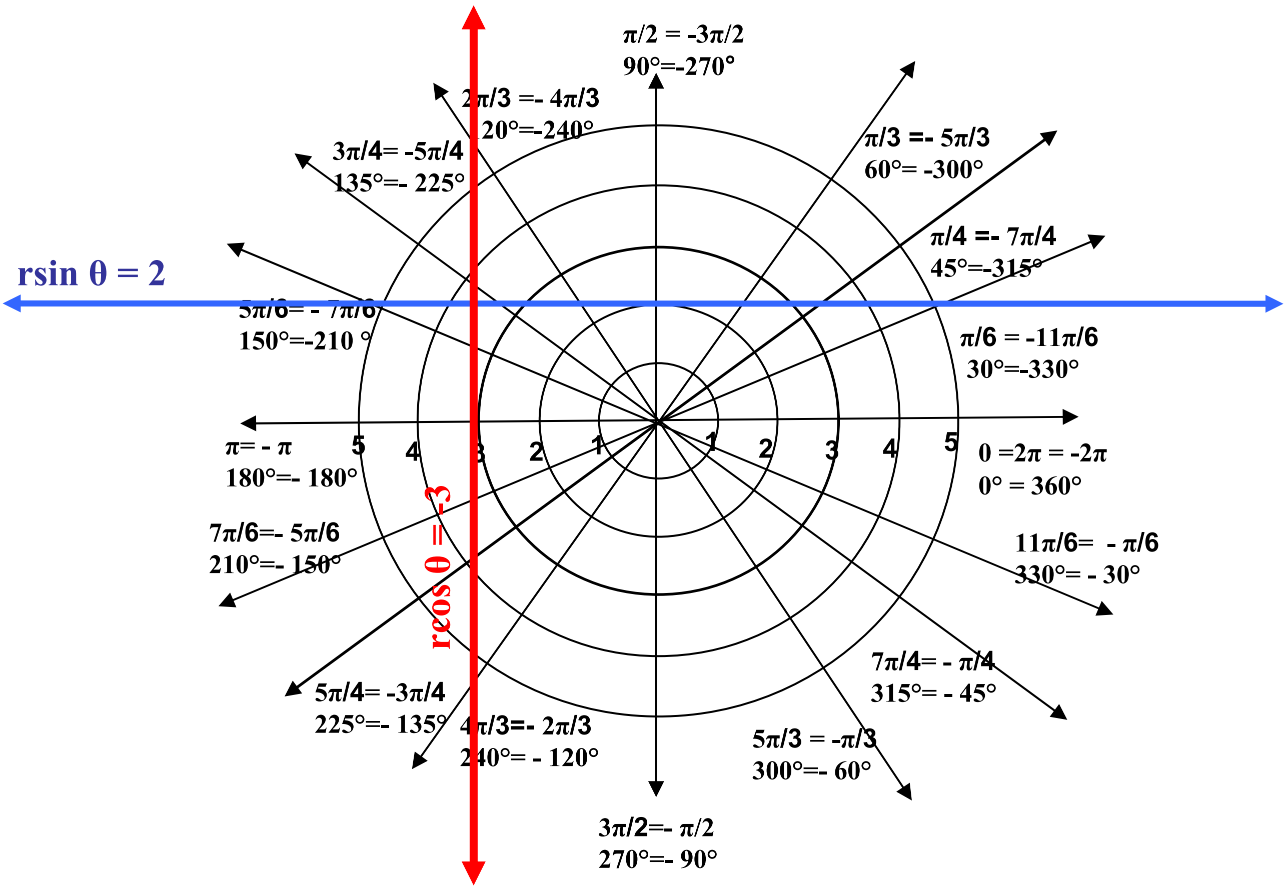
This is the graph of all points for any value of r along the ray at $\theta = \pi/4$, with positive r values along the terminal side and negative r values point in the opposite direction. Converting to rectangular coordinates we get: $y/x = \tan^{-1} \pi/4 = 1$

So multiplying both sides by x gives $y = x$.



Now you do #3 on p.343

Example 3/ Example 5: Identify and graph the equations rsin θ = 2 and rcos θ = -3 This one is not as easily identifiable when only looking at polar coordinates. But if we convert to rectangular coordinates it is simple. x = rcos θ and y = rsin θ, so substituting in y for rsin θ gives *y = 2* and substituting x for rcos θ gives *x = -3*. On a polar grid, rcos θ = -3 is 3 units on the *opposite* side of the polar axis.



<u>For future reference:</u>

Any polar equation in the form rsin θ = a is a horizontal line going through the rectangular coordinate point (0,a).
And rcos θ = a is a vertical line going through the rectangular coordinate point (a,0).

GRAPING POLAR EQUATIONS USING A GRAPHING CALCULATOR

For the TI-83Plus see p. 166 -175 in
http://education.ti.com/guidebooks/graphing/83p/83m$book-eng.pdf

Step 1: Solve the equation for r in terms of θ

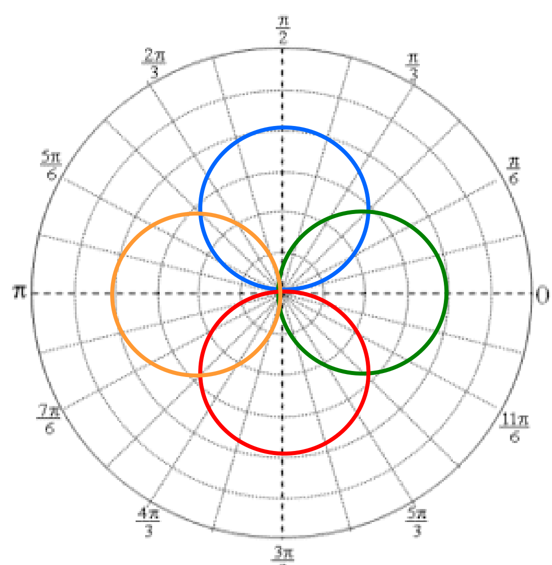
Step 2: Change MODE to Radian (third line down, first entry, though Degree mode will be easier when making tables) and also change to Pol (fourth line down, third entry from the left). Notice that the viewing window is different now. In addition to setting Xmin, Xmax, Xscl and so forth, the viewing window in polar mode requires setting minimum and maximum values for θ and an increment setting for θ (θstep). Set the ZOOM to 6:ZStandard. This automatically sets θmin to 0, θmax to 2π and θstep to π/24.

Step 3: Press Y= (notice it now shows r_1 = now instead of Y_1) and enter the expression involving θ that you found in Step 1 and press GRAPH.

Step 4: If you cannot see the graph well, select ZOOM 0:ZFit and then ZOOM 5:ZSquare so that the graph is not distorted.

POLAR CIRCLES

Equation	Description
$r = 2a\sin\theta$	Circle; radius a; center at rectangular coordinate *(0,a)*
$r = -2a\sin\theta$	Circle; radius a; center at rectangular coordinate *(0,-a)*
$r = 2a\cos\theta$	Circle; radius a; center at rectangular coordinate *(a,0)*
$r = -2a\cos\theta$	Circle; radius a; center at rectangular coordinate *(-a,0)*



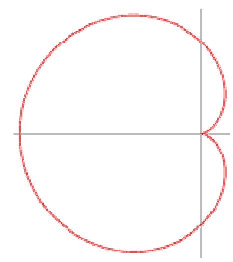
If you have a polar equation in one of these forms, you don't have to convert it to rectangular coordinates to graph it.

SYMMETRY IN POLAR COORDINATES

Tests for Symmetry

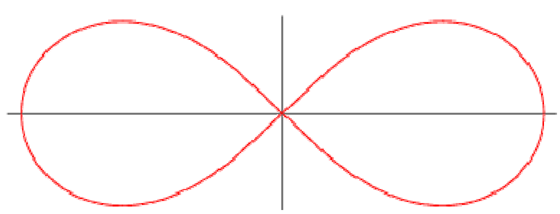
Symmetry with Respect to the Polar Axis (x-axis)

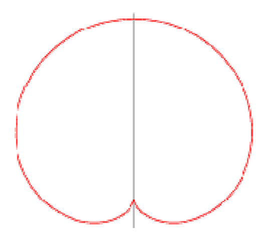
In a polar equation, replace θ by $-\theta$ and if an equivalent equation results, the graph is symmetric with respect to the polar axis (positive x-axis). [Recall that $\cos(\theta) = \cos(-\theta)$].



Symmetry with Respect to the Line $\theta = \pi/2$ (y-axis)

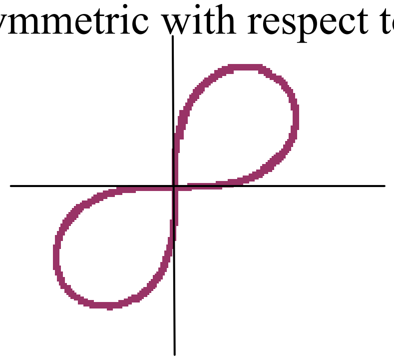
In a polar equation, replace θ by $\pi - \theta$. If an equivalent equation results, the graph is symmetric with respect to the line $\theta = \pi/2$ [Recall that $\sin(\theta) = \sin(\pi-\theta)$]

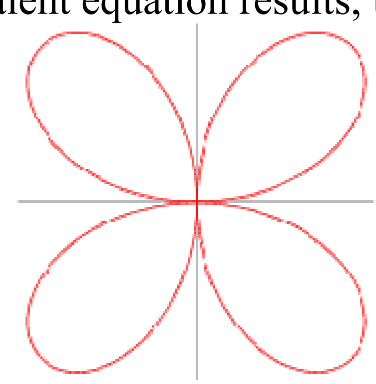




Symmetry with Respect to the Pole (origin)

In a polar equation, replace r by -r. If an equivalent equation results, the graph is symmetric with respect to the pole.

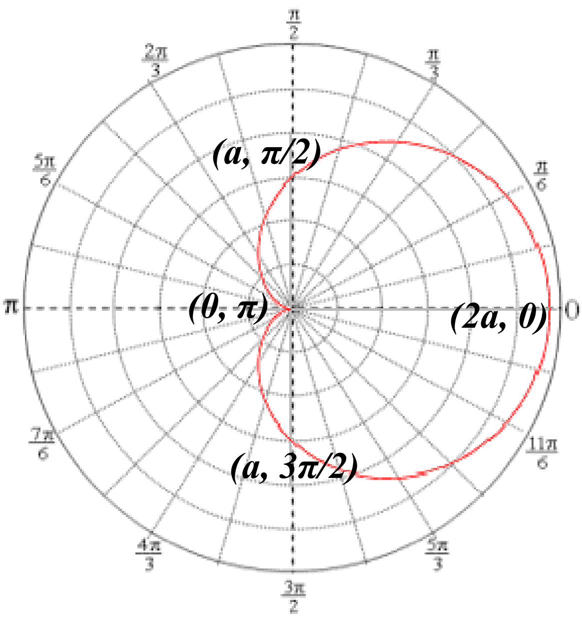




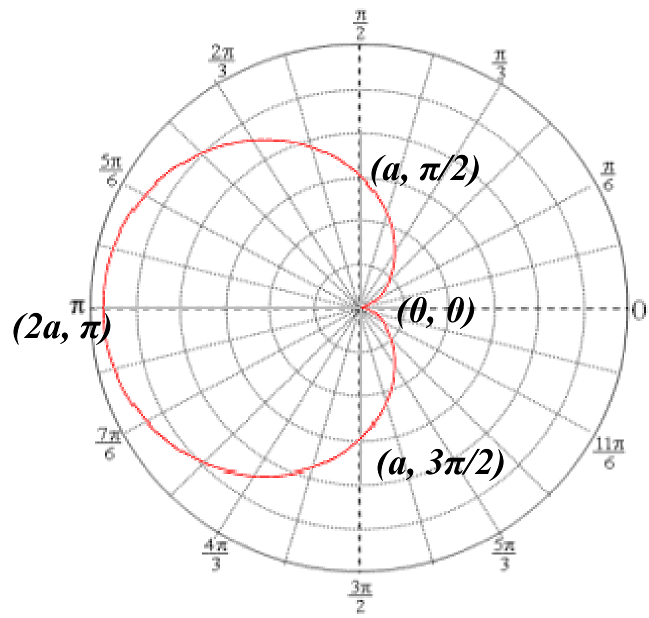
Cardioid

A cardioid is a curve given by one of the following polar equations:

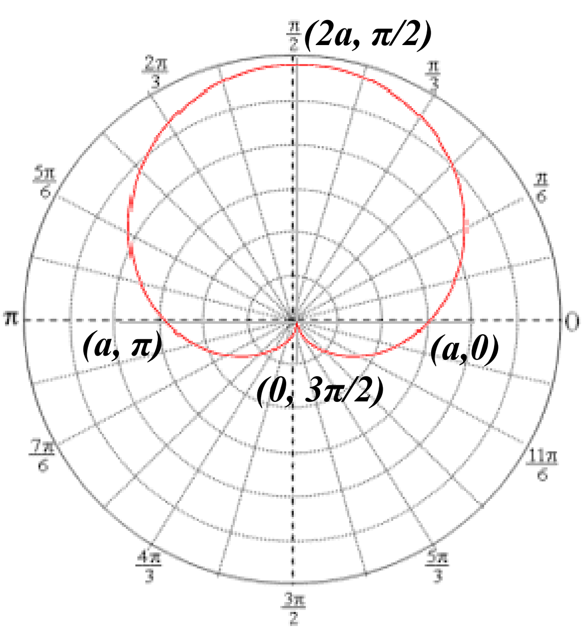
$r = a(1 + \cos\theta)$



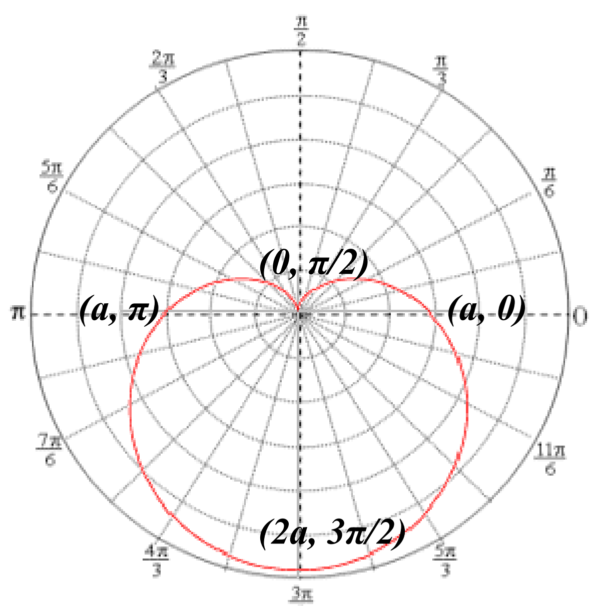
$r = a(1 - \cos\theta)$



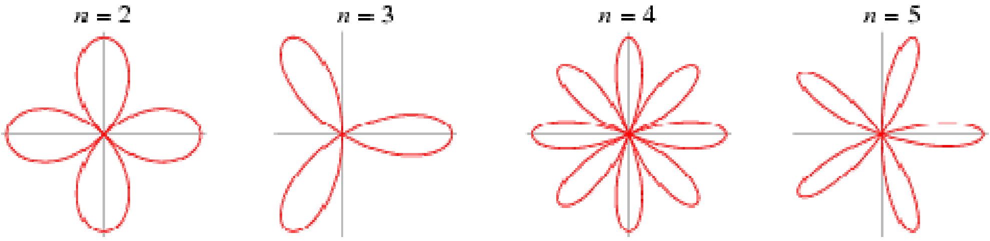
$r = a(1 + \sin\theta)$



$r = a(1 - \sin\theta)$



Rose



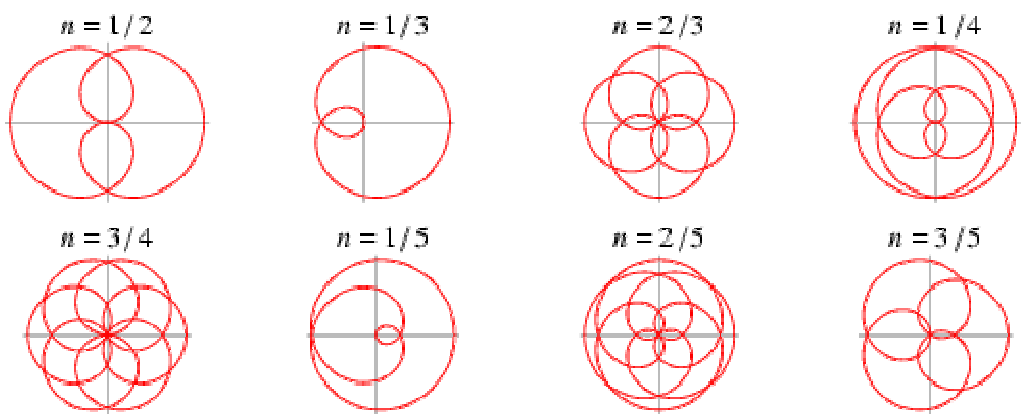
A curve which has the shape of a petalled flower. This curve was named rhodonea by the Italian mathematician Guido Grandi between 1723 and 1728 because it resembles a rose (MacTutor Archive). The polar equation of the rose is

$$r = a \sin(n\theta)$$

or

$$r = a \cos(n\theta)$$

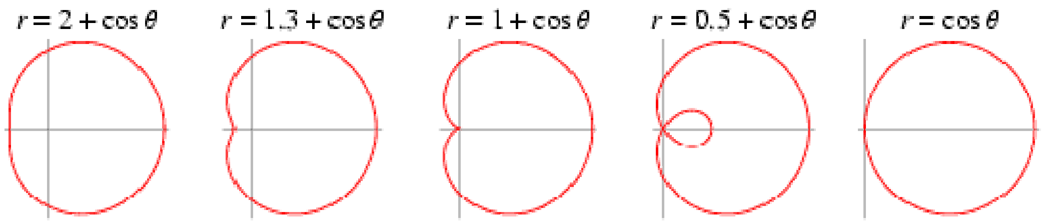
If n is odd, the rose is n-petalled. If n is even, the rose is 2n-petalled.



Eric W. Weisstein. "Rose." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/Rose.html

Limaçon

$b \geq 2a$



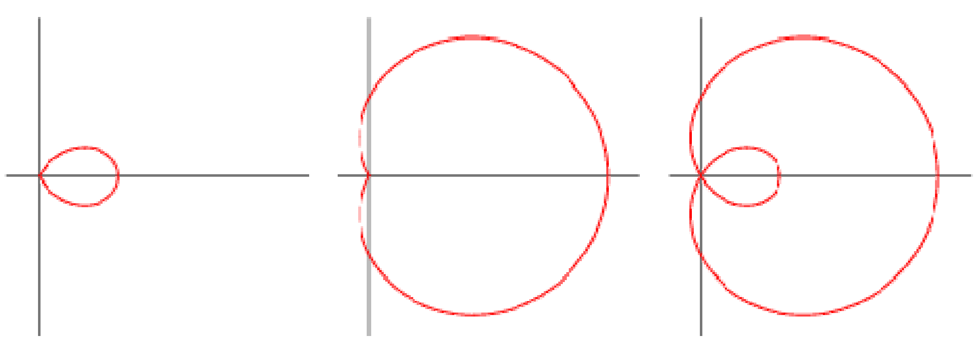
$2a > b > a$

The limaçon is a polar curve of the form

(1) $r = b + a\cos\theta$

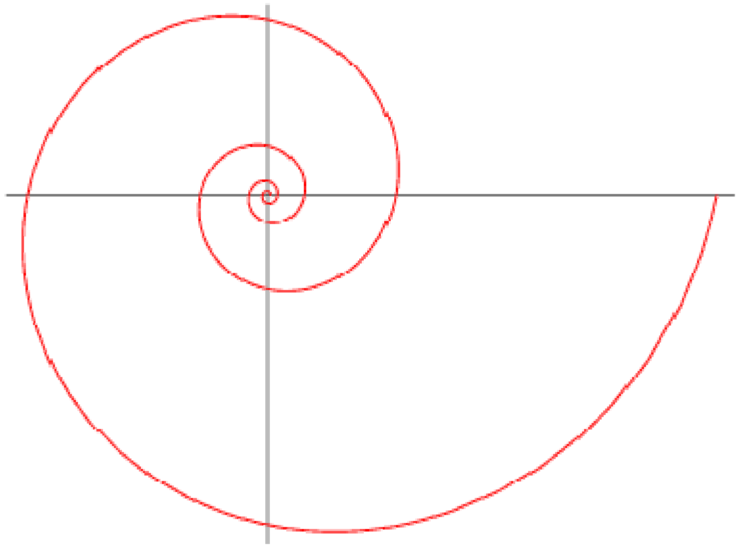
also called the limaçon of Pascal. It was first investigated by Dürer, who gave a method for drawing it in *Underweysung der Messung* (1525). It was rediscovered by Étienne Pascal, father of Blaise Pascal , and named by Gilles-Personne Roberval in 1650 (MacTutor Archive). The word "limaçon" comes from the Latin *limax*, meaning "snail."

If $b \geq 2a$, the limaçon is convex. If $2a > b > a$, the limaçon is dimpled. If $b=a$, the limaçon degenerates to a cardioid. If $b < a$, the limaçon has an inner loop. If , it is a trisectrix (but *not* the Maclaurin trisectrix).



Eric W. Weisstein. "Limaçon." From *MathWorld*--A Wolfram Web Resource.
http://mathworld.wolfram.com/Limacon.html

Logarithmic Spiral



The logarithmic spiral is a spiral whose polar equation is given by

$$r = ae^{b\theta}$$

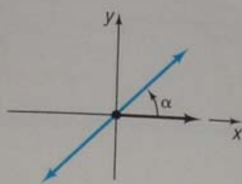
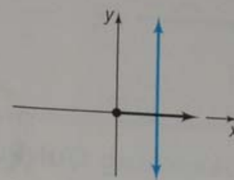
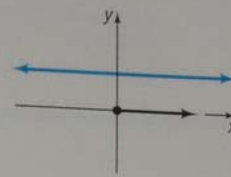
where   is the distance from the origin,   is the angle from the *x*-axis, and ***a*** and ***b*** are arbitrary constants.

Eric W. Weisstein. "Logarithmic Spiral." From *MathWorld*--A Wolfram Web Resource.

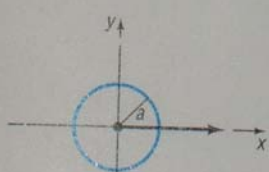
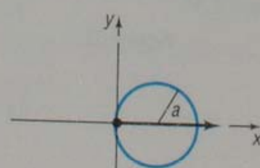
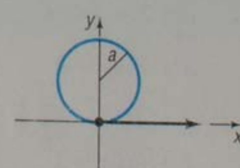
http://mathworld.wolfram.com/LogarithmicSpiral.html

POLAR EQUATIONS AND GRAPHS
P. 341-342

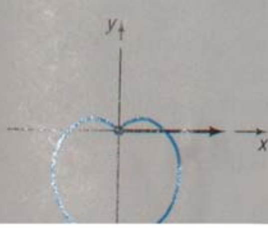
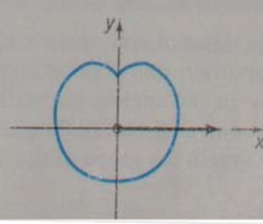
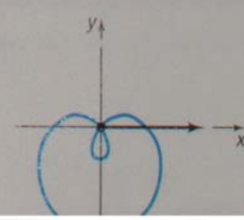
Lines

Description	Line passing through the pole making an angle α with the polar axis	Vertical line	Horizontal line
Rectangular equation	$y = (\tan \alpha)x$	$x = a$	$y = b$
Polar equation	$\theta = \alpha$	$r \cos \theta = a$	$r \sin \theta = b$
Typical graph			

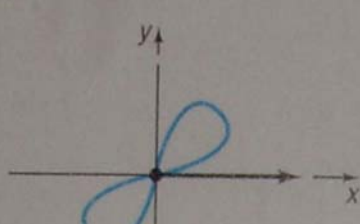
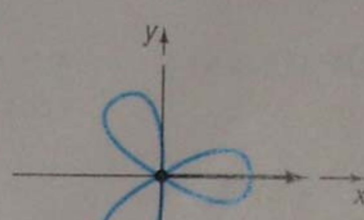
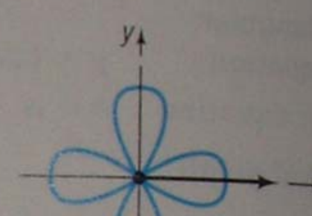
Circles

Description	Center at the pole, radius a	Passing through the pole, tangent to the y-axis, center on the x-axis, radius a	Passing through the pole, tangent to the x-axis, center on the y-axis, radius a
Rectangular equation	$x^2 + y^2 = a^2, \quad a > 0$	$x^2 + y^2 = \pm 2ax, \quad a > 0$	$x^2 + y^2 = \pm 2ay, \quad a > 0$
Polar equation	$r = a, \quad a > 0$	$r = \pm 2a \cos \theta, \quad a > 0$	$r = \pm 2a \sin \theta, \quad a > 0$
Typical graph			

Other Equations

Name	**Cardioid**	Limaçon without inner loop	Limaçon with inner loop
Polar equations	**r = a ± acos θ, a > 0** **r = a ± acos θ, a > 0**	$r = a \pm b \cos \theta, \quad 0 < b < a$ $r = a \pm b \sin \theta, \quad 0 < b < a$	$r = a \pm b \cos \theta, \quad 0 < a < b$ $r = a \pm b \sin \theta, \quad 0 < a < b$
Typical graph			

Other Equations

me	Lemniscate	Rose with three petals	Rose with four peta
ar equations	$r^2 = a^2 \cos(2\theta), \quad a > 0$ $r^2 = a^2 \sin(2\theta), \quad a > 0$	$r = a \sin(3\theta), \quad a > 0$ $r = a \cos(3\theta), \quad a > 0$	$r = a \sin(2\theta), \quad a >$ $r = a \cos(2\theta), \quad a >$
ical graph			

HOMEWORK

p. 344

1, 2, 7, 9,17, 19, 21, 23,31,39, 43, 37, 47

Copy this page to make your own polar graphs.